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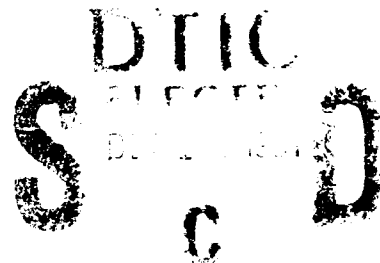
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20 November 1991

Dr. Marc J. Lipman, Scientific Officer  
Office of the Chief of Naval Research  
Department of the Navy  
Arlington, Virginia 22217-5000



Re Award N00014-85-K-0768  
*Topological Representation of Graph Isomorphism Types*

Dear Dr. Lipman:

Enclosed herewith is my final technical report and narrative. I am grateful to the ONR for its support of my research over the duration of the contract.

Sincerely yours,

Jonathan L. Gross

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## Topological Representation of Graph Isomorphism Types

The strategic plan of this research program was to develop the theory of imbedding distributions as a possible probabilistic approach to the graph isomorphism problem. It was projected that, although the much pursued goal of a practical polynomial-time test for graph isomorphism might not be achieved, the expected mathematical development in this novel attempt would provide a suitable return for the investment.

Gross and Furst constructed a hierarchy of invariants in 1985, ranging from conveniently sampled invariants, such as average genus, near the bottom to rather intricate, complete invariants much higher up. Gross and Tucker established in 1988 that the stratified graph induced by adjacency relations on imbeddings is a complete invariant. With a sampling approach, it was necessary to establish statistical distinguishability, beyond completeness itself. Gross and Chen achieved major progress on the distinguishability problem by proving in 1990 that the average genus of a graph has no essential limit points.

It was hoped that 3-connected graphs would prove to be distinguishable with lower invariants in the hierarchy, since there are tractable inductive constructions of the family of all 3-connected graphs. However, algebraic methods of Rieper established that none of the well-understood lower invariants was complete for the 3-connected simple graphs. Thus, obtaining a graph isomorphism test by this avenue seems to involve either rising in the hierarchy of invariants or identifying a nearly exhaustive class of graphs for which the lower invariants are complete.

This research has launched the distributional and enumerative theory of graph imbeddings, which along with minors (of Robertson & Seymour), is presently one of the two most active areas in topological graph theory. Other byproducts were a polynomial-time algorithm (with Furst and McGeoch) for calculating the maximum genus of a graph and a computer graphics interface (with Helfman) for designing graph imbeddings.

## Refereed Archival Journals

### 5 Papers Already Published

Hierarchy for imbedding-distribution invariants of a graph (with M. Furst), J. Graph Theory 11 (1987), 205-220.

Finding a maximum-genus graph imbedding (with M. Furst and L. McGeoch), J. ACM 35 (1988), 523-534.

Genus distribution for two classes of graphs (with M. Furst and R. Statman), J. Combinatorial Theory B 46 (1989), 22-36.

Genus distributions for bouquets of circles (with D. P. Robbins and T. W. Tucker), J. Combinatorial Theory B 47 (1989), 292-306.

Local extrema in genus-stratified graphs (with R. G. Rieper), J. Graph Theory 15 (1991), 159-171.

### 5 Papers Accepted but not yet in print

Limit points for average genus, (I): 3-connected and 2-connected simplicial graphs (with J. Chen), J. Combinatorial Theory B, to appear.

Limit points for average genus (II): 2-connected non-simplicial graphs (with J. Chen), J. Combinatorial Theory B, to appear.

Kuratowski-type theorems for average genus (with J. Chen), J. Combinatorial Theory B, to appear.

On the average genus of a graph (with E. W. Klein and R. G. Rieper), Graphs and Combinatorics, to appear.

Overlap matrices and total imbedding distributions (with J. Chen and R. G. Rieper), Discrete Mathematics, to appear.

### 4 Papers Submitted to Refereed Journals but not yet accepted

Stratified graphs (with T. W. Tucker), preprint.

Limit points for average genus (III): lower limit points (with J. Chen), preprint.

Linearly synthesizing 2-connected simplicial graphs (with J. Chen), preprint.

Lower bounds for the average genus (with J. Chen and R. G. Rieper), preprint.

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## Non-Refereed Papers

### 1 Invited Address

A probabilistic/topological approach to graph isomorphism testing, Graph Theory Notes of New York XIX (1990) (ed. J. W. Kennedy, L. V. Quintas), NY Acad. of Sciences.

### 4 Technical Reports

Extending a graph browser for topological graph theory (with J. I. Helfman), AT&T Bell Laboratories Technical Memorandum 11264-901126-10TMS, (1990).

Partitioning a graph into even subgraphs (with R. G. Rieper), preprint (1990).

Rank distributions for some adjacency hypercubes (with M. Oren and R. G. Rieper), in preparation

Adjacency hypercubes (with R. G. Rieper), in preparation.

### 1 Book Published

*Topological Graph Theory* (with T. W. Tucker), Wiley-Interscience, New York, 1987.

### 1 Chapter in Edited Volume

Burnside-Polya Counting Methods, Chapter 12 of *Applications of Discrete Mathematics* (ed. J. G. Michaels and K. H. Rosen), McGraw-Hill, 1991.

## Ph.D. Students Graduated

Bruce Abramson (1987), Asst. Prof. of Computer Science, University of Southern California

Jianer Chen (1990), Asst. Prof. of Computer Science, Texas A&M University